Mapping Structure Across Domains

Invariant Structure, Constraint, and Representation in Mathematics, Physics, and Cognition

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Abstract

In the preceding works, we developed an intuitive understanding of invariant structure under constraint and introduced a minimal structural framework describing how such structure emerges, stabilizes, and is observed. The present paper examines whether this framework applies beyond isolated mathematical examples.

We show that systems across mathematics, physics, and cognition can be described using a common schema involving configuration spaces, constraints, operator dynamics, invariant structure, and representation. Through a series of domain-specific examples, we demonstrate that similar structural patterns arise in distinct contexts, suggesting a shared underlying organization.

The goal of this work is not to assert identity across domains, but to identify structure-preserving correspondences. This establishes a foundation for understanding how mathematical descriptions extend naturally to more general systems, and prepares the ground for broader frameworks such as Quantum Collapse Geometry.

1 Introduction: From Formalism to Mapping

In earlier work, we identified a recurring pattern in mathematical systems: structure emerges under constraint and persists as invariant residue. We then introduced a minimal descriptive framework capturing this process in terms of configuration space, admissibility, operator action, invariant structure, and representation.

A natural question follows:

> Does this structural pattern extend beyond individual mathematical systems?

In this paper, we explore that question by examining how the same schema appears across multiple domains. Our aim is not to collapse distinctions between domains, but to identify common structural features and clarify what is preserved under mapping.

2 The Structural Schema

We recall the minimal framework:

$$(\Sigma, A, \Phi, I, P)$$

where:

- Σ is a configuration space,

- $A \subseteq \Sigma$ defines admissible configurations,
- $\Phi : \Sigma \to \Sigma$ is an operator,
- $I \subseteq A$ is invariant structure,
- $P : \Sigma \to O$ is a projection into observable form.

This schema captures the interaction between possibility, constraint, transformation, and observation.

3 Domain I: Elementary Mathematics

Consider rational numbers and their decimal representations.

- Σ: rational numbers,
- A: closure under division,
- Φ: multiplication by base followed by modular reduction,
- I: repeating cycles or terminating states,
- P: base-b expansion.

Repeating decimals arise from finite cycles under Φ, while terminating decimals correspond to configurations that collapse to zero under iteration.

Insight:

Representation reveals invariant cycles but does not define them.

4 Domain II: Modular Arithmetic

In modular arithmetic:

- $\Sigma = \mathbb{Z}/n\mathbb{Z}$,
- A: equivalence under modulus,
- Φ: addition or multiplication mod n,
- I: cycles, fixed points, and residue classes,
- P: sequence representation.

The system is finite, and repeated application of Φ produces periodic behavior.

Insight:

Finite constraint spaces produce cyclic invariant structure.

5 Domain III: Physical Systems (Structural Analogy)

Consider a simplified physical system:

- Σ: field configurations or state space,
- A: physical constraints (e.g., conservation laws),
- Φ: dynamical evolution,
- I: stable modes or conserved quantities,
- P: measurement or mathematical representation.

Important:

> This is a structural analogy, not an identification of physical and mathematical systems.

Insight:

> Physical structure appears as invariants under constrained dynamics.

The mapping preserves structural relationships at the level of invariance, but does not imply equivalence of underlying ontology or causal structure.

6 Domain IV: Cognition and Learning

Consider conceptual reasoning:

- Σ: possible conceptual configurations,
- A: coherence constraints,
- Φ: reasoning or refinement process,
- I: stable conceptual structures,
- P: linguistic articulation.

Understanding emerges through repeated refinement, where unstable configurations are eliminated and stable structures persist.

Insight:

> Conceptual understanding emerges as invariant structure stabilized through iterative constraint.

7 Domain V: Social and Behavioral Systems

Consider social systems:

- Σ: possible behavioral patterns,
- A: viability constraints (e.g., cooperation),
- Φ: interaction dynamics,
- I: stable norms or structures,
- P: cultural expression.

Insight:

Persistent social structures arise through selection under constraint.

8 Cross-Domain Comparison

Domain	Σ	A	Φ	I	P
Math	numbers	closure conditions	arithmetic operations	invariants	representation
Modular	residues	modulus	iteration	cycles	sequences
Physics	fields	laws	dynamics	modes	equations
Cognition	concepts	coherence	reasoning	stable ideas	language
Social	behavior	viability	interaction	norms	culture

9 What Is Preserved?

Across domains, what persists is not the objects themselves, but:

- constraint relationships,
- invariant structure,
- stability under transformation.

Key Statement:

The objects differ, but the structural pattern persists.

Not all structural similarities correspond to valid mappings, and improper identification of structure across domains can lead to misleading conclusions.

10 Limits of the Mapping

These correspondences are not identities.

- Different domains preserve different aspects of structure,
- mappings may be partial,
- analogies must be verified rather than assumed.

Caution:

> Structural similarity does not imply ontological equivalence.

11 Toward Generalization

The recurrence of this pattern suggests the possibility of a broader principle:

> Systems exhibiting constraint, transformation, and persistence may be described within a unified structural framework.

The Quantum Collapse Geometry (QCG) framework develops this idea further by modeling physical structure as arising through constraint-driven selection in relational configuration spaces. The present work does not assume that framework, but provides the structural motivation for it.

12 Conclusion: Structure Across Domains

We have shown that a common structural schema appears across multiple domains. While the objects differ, the relationships between constraint, operator action, invariant structure, and representation remain consistent.

Final Statement:

> Structured systems across domains can be understood as configurations constrained, transformed, and stabilized into invariant forms, observed through representation.